Generalized Vague Bitopological Structure Space

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Abstract

In this paper we introduce the concept of generalized vague bitopological structure space. Also, some interesting properties of compactness in generalized vague bitopological structure space are discussed. Further we discuss the properties of vague generalized bistructure bitopological space.

Keywords: generalized vague topology, generalized vague bitopology, generalized vague bitopological structure space, generalized vague compactness, vague Haudorff space.

1.Introduction

The theory of fuzzy topological spaces was introduced and developed by Chang[3]. Since then various notions in classical topology have been extended to fuzzy topological spaces by fuzzy topologies like Azad [1], Zadeh [11], Tomasz Kubiak [7, 8], Tuna Hatice Yalvac[9], Brown [2], Goguen [5]. Gau[4] et al. propose the notion of Vague sets (VSs), which allow using interval-based membership instead of using point-based membership as in fuzzy sets. The interval-based membership generalization in VSs is more expressive in capturing vagueness of data. In this paper we introduce the concept generalized vague bitopological structure space . Some interesting properties of the concepts introduced are also studied.

2.Preliminaries

Definition 2.1:[4]

A Vague set A in the universe of discourse S is a Pair (t_A, f_A) where t_A : S → [0,1] and f_A : S → [0,1] are mappings (called truth membership function and false membership function respectively) where t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and f_A(x) is a lower bound on the negation of x derived from the evidence against x and 0 ≤ t_A(x) ≤ 1- f_A(x) ≤ 1 ∀x∈ S. The set of all vague sets on X is denoted by VS(X).

Definition 2.3: [4]

A Vague set A of S is said to be contained in another Vague set B of S. That is A ⊆B, if and only if V_A(x) ≤ V_B(x). That is t_A(x)≤ t_B(x) and $1 - f_A$(x)≤ $1 - f_B$(x) ∀x ∈ S.

Definition 2.4: [4]

Two Vague sets A and B of S are equal (i.e) A = B, if and only if A⊆B and B⊆A. (i.e) V_A(x)≤ V_B(x) and V_B(x)≤ V_A(x) ∀x∈ S, which implies t_A(x) = t_B(x) and $1 - f_A$(x) = $1 - f_B$(x).

Definition 2.5 :[4]

The Union of two vague sets A and B of S with respective truth membership and false membership functions t_A, f_A and t_B, f_B is a Vague set C of S, written as C = A ∪ B, whose truth membership and false membership functions are related to those of A and B by t_C = max$\{t_A, t_B\}$ and $1 - f_C$ = max $\{1 - f_A , 1 - f_B\}$=1-min$\{f_A, f_B\}$.

Definition 2.6: [4]

The Intersection of two vague sets A and B of S with respective truth membership and false membership functions t_A, f_A and t_B, f_B is a Vague set C of S, written as C = A ∩ B, whose truth membership and false membership functions are related to those of A and B by t_C = min$\{t_A, t_B\}$ and $1 - f_C$ = min $\{1 - f_A , 1 - f_B\}$=1-max$\{f_A, f_B\}$

Definition 2.7: [4]

Let A be a Vague set of the niverse S with truth membership function t_A and false membership function f_A, for α,β ∈ [0,1] with α≤β, the (α, β) cut or Vague cut of the Vague set A is a crisp subset $A_{(\alpha,\beta)}$ of S given by $A_{(\alpha,\beta)}$ ={x∈S: V_A(x)≥ (α, β)}, (i.e) $A_{(\alpha,\beta)}$ = {x∈S: t_A(x)≥α and 1-f_A(x)≥ β}

Definition 2.8: [4] The α-cut, A_α of the Vague set A is the (α,α) cut of A and hence it is given by A_α = { x∈S : V_A(x) ≥ α}.

3. Generalized vague bitopological structure space

Definition 3.1:

Let (X, V_{τ_i},V_{τ_j}) be a vague bitopological space. A vague set A in (X, V_{τ_i},V_{τ_j}) is said to be a (i, j) - generalized vague closed set if τ_j −vcl(A) ⊆ G whenever A ⊆ G and G is a $\tau_i$vague open set. The complement of a (i, j) - generalized vague closed set is called a (i, j) - generalized vague open set.

Definition 3.2:

A family G of (i, j) - generalized vague open sets in a vague bitopological space (X, V_{τ_i},V_{τ_j}) is said to be (i, j) - generalized vague structure on X if it satisfies the following axioms:

1. $0_\sim$, $1_\sim$ ∈ G
2. A_1∩ A_2∈G, for any A_1, A_2∈G
3. ∪ A_i∈G, for any arbitrary family of generalized vague open sets $\{A_i$: i ∈ J$\}$⊆ G.

The pair (X, G) is called a (i, j) - generalized vague structure space. The members of G are called (i, j) - generalized vague open sets. The complement of a (i, j) - generalized vague structure open set is a (i, j) - generalized vague structure closed set.

Definition 3.3:

Let (X, G_1, G_2) be a (i, j) - vague generalized bi-structure space. A vague set A is called generalized vague G_1G_2 open, generalized vague G_1G_2 closed provided A in $G_1 \cup G_2$, $\bar{A}$ in $G_1 \cup G_2$ respectively.

Definition 3.4:

Let (X, G_1, G_2) be a (i, j) - vague generalized bi-structure space. A collection U of vague set in X is called a (i, j) - generalized vague structure semi open ((i, j) - generalized vague structure pairwise pen) if U $\subseteq G_1 \cup G_2$ (U $\subseteq G_1 \cup G_2$ and U contains a non-zero (i, j) - generalized vague structure open set in G_1 and non-zero (i, j) -generalized vague structure open set in G_2). A collection C is called a (i, j) -generalized vague structure semi closed ((i, j) -generalized vague structure pairwise closed) if U = {$\bar{A}$: A ∈ C} is a (i, j) - generalized vague structure semi open ((i, j) - generalized vague structure pairwise open).

Definition 3.5:

Let (X, G_1, G_2) be a (i, j) -vague generalized bi-structure space. A collection U of vague sets in X is called a vague (α, β) shading if for each x ∈ X, there exists a vague set A = <x, [t_A, 1-f_A]> in U with t_A(x) > α and 1-f_A(x) > β where 0 ≤ α < 1, 0 ≤ β < 1. A subcollection V of U that is also a vague (α, β) shading is called a vague (α, β) sub shading.

Definition 3.6:

Let (X, G_1, G_2) be a (i, j) - vague generalized bi-structure space. A collection U of vague sets in X is called a (i, j) - generalized vague structure semi (α, β) shading ((i, j) - generalized vague structure pairwise (α, β) shading) if for each x ∈ X, there exists a (i, j) - generalized vague structure semi open set A = <x, [t_A, 1-f_A]> in U such that t_A(x) > α and 1-f_A(x) > β , where 0 ≤ α < 1, 0 ≤ β < 1 and U is (i, j) - generalized vague structure semi open ((i, j) - generalized vague structure pairwise open).

Definition 3.7:

Let (X, G_1, G_2) be a (i, j) - vague generalized bi-structure space. A collection C with C $\subseteq G_1 \cup G_2$ is said to be a (i, j) - generalized vague structure semi open cover ((i, j) -generalized vague structure pairwise open cover) if $\bigcup C = 1_\sim$ and C is (i, j) - generalized vague semi open ((i, j) - generalized vague structure pairwise open).

Definition 3.8:

A vague generalized bi-structure (X, G_1 , G_2) is said to be a (i, j) - generalized vague structure semi compact space ((i, j) - generalized vague structure pairwise compact space) if every (i, j) -generalized vague structure semi open cover ((i, j) - generalized vague structure pairwise open cover) has a finite subcover.

Remark 3.9:

When we say a (i, j) - vague generalized bi-structure space (X, G_1 , G_2) has a particular property, without referring specially to G_1 or G_2, we shall mean that G_1 and G_2 have the property; for instant (X, G_1 , G_2) is said to be (i, j) - generalized vague structure compact if both (X, G_1) and (X,G_2) are (i, j) - generalized vague structure compact.

Definition 3.10:

Let (X, G_1) and (Y, G_2) be any two (i, j) - generalized vague structure spaces.
A map ψ : (X, G_1) → (Y, G_2) is (i, j) - generalized vague structure continuous if inverse image of every (i, j) - generalized vague structure open set in (Y,G_2) is a (i, j) - generalized vague structure open set in (X, G_1).

Proposition 3.11:

A (i, j) - vague generalized bi structure space (X, G_1 , G_2) is (i, j) - generalized vague structure semi (α, β) compact space if and only if (X, G_1 , G_2) is a (i, j) - generalized vague structure pairwise (α, β) compact space and (i, j) - generalized vague structure (α, β) compact space.

Proof:

Let U be a (i, j) - generalized vague structure pairwise (α, β) shading of X. Then U is a (i, j) - generalized vague structure semi (α, β) shading of (X, G_1 , G_2) and so it has finite (i, j) - generalized vague structure (α, β) sub shading. Conversely, Let U be a (i, j) - generalized vague structure semi (α, β) shading of X. Then either U ⊆ G_1 or U ⊆ G_2 or U is a (i, j) - generalized vague structure paiwise (α, β) shading of X. In either case U has a finite (i, j) - generalized vague structure (α,β) sub shading.

Definition 3.12:

A vague set A = <x, [t_A, 1-f_A]> in a (i, j) - generalized vague structure space (X, G) is said to be a (i, j) - generalized vague structure compact space if for every family U ⊆ G such that t_A ≤ sup{t_B: B = <x, [t_B, 1-f_B]> ∈ U}, 1- f_A ≤ sup{$1 - f_B$: B = <x, [t_B, 1-f_B]> ∈ U}and for every ε > 0, there exists a finite subfamily U_ε ⊆ U such that t_A - ε ≤ sup{t_B: B = <x, [t_B, 1-f_B]> ∈ U_ε } , $(1 - f_A)$ - ε ≤ sup{$1 - f_B$: B = <x, [t_B, 1-f_B]> ∈ U_ε }.

Definition 3.13:

A vague set A = <x, [t_A, 1-f_A]> in a (i, j) - vague generalized bistructure space (X, G_1 , G_2) is said to be a (i, j) - generalized vague structure semi ((i, j) - generalized vague structure pairwise) compact space if for every family U of (i, j) - generalized vague structure semi open ((i, j) - generalized vague structure pairwise open) sets, such that t_A ≤ sup{t_B: B =<x,

$[t_B$, 1-$f_B]$>∈ U},$1-f_A \leq \sup\{1-f_B$:B ∈ U} and for every ε > 0, there exist a finite subfamily $U_\varepsilon \subseteq$ U such that t_A - ε ≤ sup{t_B: B∈ U} $(1-f_A)$ - ε ≤ sup{$1-f_B$: B ∈ U_ε}.

Proposition 3.14:

Let (X, G_1 , G_2) and (Y, L_1 , L_2) be any two (i, j) - vague generalized bistructure spaces and ψ : (X, G_1 , G_2) → (Y, L_1 , L_2) be a (i, j) - generalized vague structure continuous surjection.

Proof:

1. If (X, G_1 , G_2) is a (i, j) - generalized vague structure pairwise (α, β) compact space, then (Y, L_1 , L_2) is a (i, j) - generalized vague structure pairwise (α, β) compact space.
2. If (X, G_1 , G_2) is a (i, j) - generalized vague structure semi (α, β) compact space , then (Y, L_1 , L_2) is a (i, j) - generalized vague structure semi (α, β) compact space.

Proof:

1. Let U be a (i, j) - generalized vague structure pairwise (α, β) shading of Y. Then ψ^{-1}(U) = {ψ^{-1}(A): A ∈ U} is a (i, j) - generalized vague structure pairwise (α, β) shading of X because if x ∈ X, then ψ(x) ∈ Y, so there exists A = <x, $[t_A$, 1-$f_A]$> ∈ U such that t_A (ψ(x)) > α and $1-f_A$ (ψ(x)) > β. That is, $\psi^{-1}(t_A$(x)) > α and $\psi^{-1}(1-f_A$(x)) > β. Hence {ψ^{-1}(A) : A ∈ U} has a finite (i, j) - generalized vague structure (α, β) subshading {$\psi^{-1}(A_i)$: i = 1,2, …..n}. Now {A_i : i = 1, 2, …..n} is a finite (i, j) - generalized vague structure (α, β) subshading of U because if y ∈Y, then y = f(x) for some x ∈ X thus there exists j such that $\psi^{-1}(\, t_{A_j})$(x) > α and $\psi^{-1}($ $1-f_{A_j}$)(x) > β. This implies that t_{A_j}(ψ(x)) = t_{A_j}(y) > α and $1-f_{A_j}$(ψ(x)) = $1-f_{A_j}$(y) > β. Hence (Y, L_1 , L_2) is a (i, j) - generalized vague structure pairwise (α, β) compact space.
2. Similar to the proof of (1).

Proposition 3.15:

Let (X, G_1 , G_2) and (Y, L_1 , L_2) be any two (i, j) - vague generalized vague bistructure spaces and f : (X, G_1 , G_2) → (Y, L_1 , L_2) be a (i, j) - generalized vague structure continuous surjection.

1. If (X, G_1 , G_2) is a (i, j) - generalized vague structure semi compact space, then (Y, L_1 , L_2) is a (i, j) - generalized vague structure semi compact space.
2. If (X, G_1 , G_2) is a (i, j) - generalized vague structure pairwise compact space, then (Y, L_1 , L_2) is a (i, j) - generalized vague structure pairwise compact space.

Proof:

1. Let U be a family of (i, j) - generalized vague structure semi open sets in Y. Then ψ^{-1}(U) = {ψ^{-1}(B) : B = <x, $[t_B$, 1-$f_B]$> ∈ U} is a (i, j) - generalized vague structure semi open sets of X because if x ∈ X, then ψ(x) ∈ Y, so there exists A = <x, $[t_A$, 1-

f_A]> ∈ U and let ε > 0 such that $t_A \leq \sup\{t_B : B \in U\}$ and $1 - f_A \leq \sup\{1 - f_B : B \in U\}$. That is, $\psi^{-1}(t_A) \leq \sup\{t_B : B \in U\}$ and $\psi^{-1}(1 - f_A) \leq \sup\{1 - f_B : B \in U\}$. Hence ψ^{-1}(U) ={ψ^{-1}(B) : B =<x, [t_B, 1-f_B]>∈U} has a finite subfamily {$\psi^{-1}(B_i)$:$B_i \in U_\varepsilon$, i = 1,2,3….n}. Now, {B_i : i = 1,2,3….n} is a finite subfamily of $U_\varepsilon \subseteq$ U because if y ∈ Y, then y = ψ(x) for some x ∈ X. Thus there exists j such that $\psi^{-1}(t_A)$ - ε ≤ sup{$\psi^{-1}(t_{B_j})$: $B_j \in U_\varepsilon$, j = 1,2,3….n}, $\psi^{-1}(t_A)$ - ε ≤ sup{$\psi^{-1}(t_{B_j})$: $B_j \in U_\varepsilon$, j = 1,2,3….n}, $\psi^{-1}[(1 - f_A)]$ - ε ≤ sup{$\psi^{-1}(1 - f_{B_j})$: $B_j \in U_\varepsilon$, j = 1,2,3….n}. This implies that t_A(ψ(x)) - ε = t_A(y)) - ε ≤ sup{ t_{B_j}: $B_j \in U_\varepsilon$, j = 1,2,3….n}, $(1 - f_A)$(ψ(x)) - ε = $(1 - f_A)$(y)) - ε ≤ sup{ $1 - f_{B_j}$: $B_j \in U_\varepsilon$, j = 1,2,3….n}. Thus, (Y, L_1 , L_2) is a (i, j) - generalized vague structure semi compact space.
2. Similar to the proof of (1).

Definition 3.16:

A (i, j) - vague generalized bistructure space (X, G_1 , G_2) is a (i, j) - generalized vague structure semi constant ((i, j) - generalized vague structure pairwise constant) compact space provided that each vague constant map from X into I is a (i, j) - generalized vague structure semi compact space ((i, j) - generalized vague structure pairwise compact space).

Proposition 3.17:

Let (X, G_1 , G_2) and (Y, L_1 , L_2) be any two (i, j) - vague generalized bistructure spaces and ψ : (X, G_1 , G_2) → (Y, L_1 , L_2) be a (i, j) - generalized vague structure continuous surjection.

1. If (X, G_1 , G_2) is a (i, j) - generalized vague structure pairwise constant compact space, then (Y, L_1 , L_2) is a (i, j) - generalized vague pairwise constant compact space.
2. If (X, G_1 , G_2) is a (i, j) - generalized vague structure semi constant compact space, then (Y, L_1 , L_2) is a (i, j) - generalized vague structure semi constant compact space.

Proof:

1. Let E = [t_E, 1-f_E] be a vague constant map in Y and let U be a family of (i, j) - generalized vague structure pairwise open sets in Y, such that $t_E \leq \sup\{t_B : B \in U\}$, $1 - f_E \leq \sup\{1 - f_B : B \in U\}$. We have to show that for ε > 0, there exists a finite subfamily $U_\varepsilon \subseteq$ U such that t_E - ε ≤ sup{t_{B_i} : $B_i \in U_\varepsilon$, i = 1,2,3,……n}, $(1 - f_E)$ - ε ≤ sup{$1 - f_{B_i}$: $B_i \in U_\varepsilon$, i = 1,2,3,……n}. Since ψ is a (i, j) - generalized vague structure continuous, ψ^{-1}(U) = {ψ^{-1}(B) : B ∈ U} is a family of (i, j) - generalized vague structure pairwise open sets in X such that $t_E \leq \sup\{\psi^{-1}(t_{B_i}): B_i \in U_i\}$, $1 - f_E \leq \sup\{\psi^{-1}(1 - f_{B_i}): B_i \in U_i\}$ because $\psi^{-1}(t_A)$(x) = t_A(ψ(x)). Since (X, G_1 , G_2) is a (i, j) - generalized vague structure pairwise constant compact, there exists a finite subfamily {$\psi^{-1}(B_i)$: $B_i \in U_\varepsilon$, i = 1,2,3,…..n}, such that t_E - ε ≤ sup{$\psi^{-1}(t_{B_i})$: $B_i \in U_\varepsilon$, i = 1,2,3,…..n}, $(1 - f_E)$ - ε ≤ sup{$\psi^{-1}(1 - f_{B_i})$: $B_i \in U_\varepsilon$, i = 1,2,3,…..n} which

implies t_E - ε ≤ sup{t_{B_i} : B_i∈ $U_ε$, i = 1,2,3,…..n}, $(1 - f_E)$ - ε ≤ sup{$1 - f_{B_i}$: B_i∈ $U_ε$, i = 1,2,3,…..n}. Hence (Y, L_1 , L_2) is a (i, j) - generalized vague structure pairwise constant compact space.

2. Similar to the proof of (1).

4 . Generalized vague compact open structure bitopological space

Definition 4.1:

Let (X, V_{τ_i},V_{τ_j}) be a vague bitopological space. A vague set A in (X, V_{τ_i},V_{τ_j}) is said to be (i, j) - generalized vague closed if τ_j − Vcl(A) ⊆ G whenever A ⊆ G and G is τ_i − vague open. The complement of a (i, j) - generalized vague closed set is a (i, j) - generalized vague open set.

Definition 4.2:

Let (X, V_{τ_i},V_{τ_j}) be a vague bitopological space and A be a vague set in X. Then (i, j) - vague generalized closure ((i, j) - VGcl for short) and (i, j) - vague generalized vague interior ((i, j) - VGint for short) of A are defined by,

1. (i, j) - VGcl(A) = ∩ {G : G is a (i, j) - generalized vague closed set in X and A ⊆ G}
2. (i, j) - VGint(A) = ∪ {G : G is a (i, j) - generalized vague open set in X and A ⊇ G}.

Definition 4.2:

Let (X, V_{τ_i},V_{τ_j}) be a vague bitopological space. If a family {G_i : i ∈ J} of (i, j) - generalized vague open sets in (X, V_{τ_i},V_{τ_j}) satisfies the condition $\bigcup_{i \in J} G_i = 1_\sim$, then it is called a (i, j) - generalized vague open cover of (X, V_{τ_i},V_{τ_j}). A finite subfamily of a (i, j) - generalized vague open cover {G_i : i ∈ J} of (X, V_{τ_i},V_{τ_j}), which is also a (i, j) - generalized vague open cover of (X, V_{τ_i},V_{τ_j}) is called a finite subcover.

Definition 4.3:

A vague bitopological space (X, V_{τ_i},V_{τ_j}) is called a (i, j) - generalized vague compact space if every (i, j) - generalized vague open cover of (X, V_{τ_i},V_{τ_j}) has a finite subcover.

Definition 4.4:

Let (X, V_{τ_i},V_{τ_j}) be a vague bitopological space and A be a vague set A in (X, V_{τ_i},V_{τ_j}). If a family {G_i : i ∈ J} of (i, j) -generalized vague open sets in (X, V_{τ_i},V_{τ_j}) satisfies the condition A ⊆ $\bigcup_{i \in J} G_i$, then it is called a (i, j) -generalized vague open cover of A. A finite subfamily of a (i, j) - generalized vague open cover of A, which also covers A is called a finite subcover of A.

Definition 4.5:

A vague set A in a vague bitopological space (X, $V_{\tau_i,}V_{\tau_j}$) is said to be a (i, j) - generalized vague compact if every (i, j) - generalized vague open cover of A has a finite subcover.

Definition 4.6:

Let X be a nonempty set. If r ∈ I_0, s ∈ I_1 are fixed real number, such that r + s ≤ 1 then the vague set $x_{r,s}$ is called a vague point (VP for short) in X given by

$x_{r,s}\ (x_p) = \begin{cases} [r, 1-s], if\ x = x_p \\ [0,1],\ \ if\ x \neq x_p \end{cases}$ for x_p ∈ X is called the support of $x_{r,s}$, where r denotes the degree of membership value and s is the degree of non-membership value of $x_{r,s}$.

A vague point $x_{r,s}$ is said to belong to a vague set A if r ≤ t_A(x) and 1-s ≤ $1 - f_A$(x)

Definition 4.7:

Let (X, $V_{\tau_i,}V_{\tau_j}$) and (Y, $V_{\sigma_i,}V_{\sigma_j}$) be any two vague bitopological spaces. A mapping ψ : (X, $V_{\tau_i,}V_{\tau_j}$) → (Y, $V_{\sigma_i,}V_{\sigma_j}$) is (i, j) - generalized vague continuous at a vague point $x_{r,s}$ of X if every (i, j) - generalized vague open set V in Y and ψ($x_{r,s}$) ∈ V, there exists a (i, j) - generalized vague open set U in X and $x_{r,s}$ ∈ U such that ψ(U) ⊆ V.

Definition 4.8:

Let (X, $V_{\tau_i,}V_{\tau_j}$) and (Y, $V_{\sigma_i,}V_{\sigma_j}$) be any two vague bitopological spaces and let ψ : (X, $V_{\tau_i,}V_{\tau_j}$) → (Y, $V_{\sigma_i,}V_{\sigma_j}$) be a mapping. Then ψ is said to be (i, j) - generalized vague open if image of each (i, j) - generalized vague open set U in (X, $V_{\tau_i,}V_{\tau_j}$) is a (i, j) - generalized vague open set ψ(U) in (Y, $V_{\sigma_i,}V_{\sigma_j}$).

Definition 4.9:

Let (X, $V_{\tau_i,}V_{\tau_j}$) and (Y, $V_{\sigma_i,}V_{\sigma_j}$) be any two vague bitopological spaces. A mapping ψ : (X, $V_{\tau_i,}V_{\tau_j}$) → (Y, $V_{\sigma_i,}V_{\sigma_j}$) is said to be (i, j) - generalized vague homeomorphism if ψ is bijective, (i, j) - generalized vague continuous and (i, j) - generalized vague open.

Definition 4.10:

A vague bitopological space (X, $V_{\tau_i,}V_{\tau_j}$) is said to be (i, j) - generalized vague Hausdorff space or T_2 space if for any two distinct vague points $x_{r,s}$ and $x_{u,v}$, there exist (i, j) - generalized vague open sets U and V, such that $x_{r,s}$ ∈ U, $x_{u,v}$ ∈ V and U ∩ V = $0_{\sim}$.

Definition 4.11:

A vague bitopological space (X, $V_{\tau_i,}V_{\tau_j}$) is said to be (i, j) - generalized vague locally compact space if for every vague point $x_{r,s}$, there exists a (i, j) - generalized vague open set G, such that $x_{r,s}$ ∈G and G is (i, j) - generalized vague compact. That is each (i, j) - generalized vague open cover of G has a finite subcover.

Remark 4.12:

A (i, j) - generalized vague compact subspace of a (i, j) - generalized vague Hausdorff space is (i, j) - generalized vague closed.

Proposition 4.13:

A (i, j) - generalized vague Hausdoff bitopological space (X, $V_{\tau_i,}V_{\tau_j}$) the following conditions are equivalent.

1. (X, $V_{\tau_i,}V_{\tau_j}$) is (i, j) - generalized vague locally compact.
2. For each vague point $x_{r,s}$, there exist a (i, j) - generalized vague open set G in X such that $x_{r,s}$ ∈ G and (i, j) - VGcl(G) is (i, j) - generalized vague compact.

Proof:

(i)⇒(ii) By hypothesis for each vague point $x_{r,s}$ in X, there exists a (i, j) - generalized vague open set G, such that $x_{r,s}$ ∈ G and G is (i, j) - generalized vague compact. Since X is (i, j) - generalized vague Hausdorff space, by Remark 4.13 G is (i, j) - generalized vague closed, thus G = (i, j) - VGcl(G). Hence, $x_{r,s}$ ∈ G and (i, j) - VGcl(G) is (i, j) - generalized vague compact.

(ii)⇒(i) Similar to the proof of above.

Proposition 4.14:

Let (X, $V_{\tau_i,}V_{\tau_j}$) be a generalized vague Hausdorff topological space. Then (X, $V_{\tau_i,}V_{\tau_j}$) is (i, j) -generalized vague locally compact on a vague point $x_{r,s}$ in X if and only if for every (i, j) - generalized vague open set G containing $x_{r,s}$, there exists a (i, j) -generalized vague open set V, such that $x_{r,s}$ ∈ V, (i, j) - VGcl(V) is (i, j) - generalized vague compact and (i, j) -VGcl(V) ⊆ G.

Definition 4.15:

Let (X, $V_{\tau_i,}V_{\tau_j}$) and (Y, $V_{\sigma_i,}V_{\sigma_j}$) be any two vague topological spaces. A mapping T : X x Y → Y x X defined by T(x, y) = (y, x) for each (x, y) ∈ X x Y is called a vague biswitching map.

Definition 4.16:

Let (X, $V_{\tau_i,}V_{\tau_j}$) and (Y, $V_{\sigma_i,}V_{\sigma_j}$)be any two vague bitopological spaces and let Y^X = {ψ : X → Y such that ψ is (i, j) - generalized vague continuous }. Let M = {K : K is (i, j) - generalized vague compact on X} and N = { V : V is (i, j) - generalized vague open set in Y}. For any K ∈ M and V ∈ N, let $S_{K,V}$ = {ψ∈ Y^X : ψ(K) ⊆ V }. The collection of all {$S_{K,V}$: K ∈ M, V ∈ N} forms a vague structure on the class Y^X. This structure is called (i, j) -generalized vague compact open structure. The class Y^X with this structure is called a (i, j) -generalized vague compact open structure space.

Definition 4.17:

A mapping e : Y^X x X → Y defined by e (ψ,$x_{r,s}$) = ψ($x_{r,s}$) for each vague point $x_{r,s}$ ∈ X and ψ ∈ Y^X is called a (i, j) - generalized vague evaluation map.

Proposition 4.18:

Let (X, $V_{\tau_i,}V_{\tau_j}$) be a (i, j) - generalized vague locally compact Hausdorff space. Then the (i, j) - generalized vague evaluation map e: Y^X x X → Y is (i, j) - generalized vague continuous.

Proposition 4.20:

Let (Z,$V_{\omega_i,}$ V_{ω_j}) be a (i, j) - generalized vague locally compact Hausdorff space and (X, $V_{\tau_i,}V_{\tau_j}$) , (Y, $V_{\sigma_i,}V_{\sigma_j}$) be any two arbitrary vague bitopological spaces. Then a map ψ : Z x X → Y is (i, j) - generalized vague continuous if and only if $\widehat{\psi}$: X → Y^Z is (i, j) - generalized vague continuous , where $\widehat{\psi}$ is defined by ($\widehat{\psi}$($x_{r,s}$))($x_{u,v}$) = ψ($x_{u,v}$, $x_{r,s}$).

Proposition 4.21:

Let (X, $V_{\tau_i,}V_{\tau_j}$) and (Z, $V_{\omega_i,}$ V_{ω_j}) be any two (i, j) - generalized vague locally compact Hausdorff spaces. Then for any vague bitopological space (Y, V_σ), the map E : Y^{ZXX}→$(Y^Z)^X$ defined by E(ψ) = $\widehat{\psi}$ (that is E(ψ)($x_{r,s}$)($x_{u,v}$) = ψ($x_{u,v}$, $x_{r,s}$) = ($\widehat{\psi}$($x_{r,s}$))($x_{u,v}$)) for all ψ : Z x X → Y is a (i, j) - generalized vague homeomorphism.

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