Neutrosophic Generalized SPR Closed Sets

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Abstract. The purpose of this paper is to introduce and study a new class of generalized closed set, namely neutrosophic generalized SPR closed sets and neutrosophic generalized SPR open sets in neutrosophic topological spaces. Also we study the separation axioms of neutrosophic generalized SPR closed sets, namely neutrosophic $sprT_{1/2}$ space and neutrosophic $sprT^*_{1/2}$ space and their properties are discussed.

Keywords: Neutrosophic set, neutrosophic topological space, neutrosophic generalized SPR closed sets, $NsprT_{1/2}$ space and $NsprT^*_{1/2}$ space.

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1 Introduction

In 1965, Zadeh [15] introduced the notion of fuzzy sets [FS]. It shows the degree of membership of the element in a set. Later, fuzzy topological space was introduced by Chang [2] in 1968. In 1983, Atanassov [1] introduced the notion of intuitionistic fuzzy sets [IFS], where the degree of membership and degree of non-membership are discussed. Later, Intuitionistic fuzzy topological spaces was introduced by Coker [3] in 1997. Neutrality the degree of indeterminacy as an independent concept was introduced by Florentin Smarandache [4] He also defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership). In 2012, Salama A. A and Alblowi [10] introduced the concept of Neutrosophic topological space by using Neutrosophic sets. Salama A. A. [11] introduced Neutrosophic closed set and Neutrosophic continuous function. Further the basic sets like regular-open sets, semi-open sets, pre-open sets and α-open sets are introduced in Neutrosophic topological space and their properties are studied by various authors [6], [11], [13]. In this direction, we introduce and analyze a new class of neutrosophic closed set called neutrosophic generalized SPR closed sets. Also we study the separation axioms of neutrosophic generalized SPR closed sets, namely neutrosophic $sprT_{1/2}$ space and neutrosophic $sprT^*_{1/2}$ space in neutrosophic topological space.

2 Preliminaries

We recall some basic definitions that are used in the sequel.

Definition 2.1: [10]

Let Y be a non-empty set. A neutrosophic set (NS for short) A in Y is an object having the form $A = \{\langle Y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle : y \in Y\}$ where the functions $\mu_A(y)$, $\sigma_A(y)$ and $\nu_A(y)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $y \in Y$ to the set A.

Remark 2.2: [10]

A Neutrosophic set $A = \{\langle y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle : y \in Y\}$ can be identified to an ordered triple $A = \langle \mu_A(y), \sigma_A(y), \nu_A(y)\rangle$ in non-standard unit interval $]^-0, 1^+[$ on Y.

Remark 2.3: [10]

For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \nu_A\rangle$ for the neutrosophic set $A = \{\langle y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle : y \in Y\}$.

Example 2.4: [10] Every IFS A is a non-empty set in Y is obviously on NS having the form $A = \{\langle y, \mu_A(y), 1 - (\mu_A(y)+ \nu_A(y)), \nu_A(y)\rangle : y \in Y\}$. Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the NS 0_N and 1_N in Y as follows:
0_N may be defined as:
(0_1) $0_N = \{\langle y, 0, 0, 1\rangle : y \in Y\}$
(0_2) $0_N = \{\langle y, 0, 1, 1\rangle : y \in Y\}$

(0_3) $0_N = \{\langle y, 0, 1, 0\rangle: y \in Y\}$
(0_4) $0_N = \{\langle y, 0, 0, 0\rangle: y \in Y\}$

1_N may be defined as:
(1_1) $1_N = \{\langle y, 1, 0, 0\rangle: y \in Y\}$
(1_1) $1_N = \{\langle y, 1, 0, 1\rangle: y \in Y\}$
(1_1) $1_N = \{\langle y, 1, 1, 0\rangle: y \in Y\}$
(1_1) $1_N = \{\langle y, 1, 1, 1\rangle: y \in Y\}$

Definition 2.5: [10]

Let $A = \langle \mu_A, \sigma_A, \nu_A\rangle$ be a NS on Y, then the complement of the set A [C(A) for short] may be defined as three kind of complements:
(C_1) $C(A) = \{\langle y, 1-\mu_A(y), 1-\sigma_A(y), 1-\nu_A(y)\rangle: y \in Y\}$
(C_2) $C(A) = \{\langle y, \nu_A(y), \sigma_A(y), \mu_A(y)\rangle: y \in Y\}$
(C_3) $C(A) = \{\langle y, \nu_A(y), 1-\sigma_A(y), \mu_A(y)\rangle: y \in Y\}$

Definition 2.6: [10]

Let Y be a non-empty set and neutrosophic sets A and B in the form $A = \{\langle y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle: y \in Y\}$ and $B = \{\langle y, \mu_B(y), \sigma_B(y), \nu_B(y)\rangle: y \in Y\}$. Then we may consider two possible definitions for subsets $(A\subseteq B)$.
(1) $A\subseteq B \Leftrightarrow \mu_A(y) \le \mu_B(y), \sigma_A(y) \le \sigma_B(y)$ and $\mu_A(y) \ge \mu_B(y)\ \forall\ y \in Y$
(2) $A\subseteq B \Leftrightarrow \mu_A(y) \le \mu_B(y), \sigma_A(y) \ge \sigma_B(y)$ and $\mu_A(y) \ge \mu_B(y)\ \forall\ y \in Y$

Proposition 2.7: [10]

For any Neutrosophic set A, the following conditions holds:
$0_N \subseteq A$, $0_N \subseteq 0_N$
$A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition 2.8: [10]

Let Y be a non-empty set and $A = \{\langle y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle: y \in Y\}$, $B = \{\langle y, \mu_B(y), \sigma_B(y), \nu_B(y)\rangle: y \in Y\}$ are NSs. Then
(1) $A\cap B$ may be defined as:
(I_1) $A\cap B = \langle y, \mu_A(y)\wedge\mu_B(y), \sigma_A(y)\wedge\sigma_B(y)$ and $\nu_A(y)\vee\nu_B(y)\rangle$
(I_2) $A\cap B = \langle y, \mu_A(y)\wedge\mu_B(y), \sigma_A(y)\vee\sigma_B(y)$ and $\nu_A(y)\vee\nu_B(y)\rangle$
(2) $A\cup B$ may be defined as:
(U_1) $A\cup B = \langle y, \mu_A(y)\vee\mu_B(y), \sigma_A(y)\vee\sigma_B(y)$ and $\nu_A(y)\wedge\nu_B(y)\rangle$
(U_2) $A\cup B = \langle y, \mu_A(y)\vee\mu_B(y), \sigma_A(y)\wedge\sigma_B(y)$ and $\nu_A(y)\wedge\nu_B(y)\rangle$

We can easily generalize the operations of intersection and union in Definition 2.8., to arbitrary family of NSs as follows:

Definition 2.9: [10]

Let $\{A_j: j \in J\}$ be an arbitrary family of NSs in Y, then
(1) $\cap A_j$ may be defined as:
(i) $\cap A_j = \langle y, \wedge_{j\in J}\ \mu_{Aj}(y), \wedge_{j\in J}\ \sigma_{Aj}(y), \vee_{j\in J}\ \nu_{Aj}(y)\rangle$
(ii) $\cap A_j = \langle y, \wedge_{j\in J}\ \mu_{Aj}(y), \vee_{j\in J}\ \sigma_{Aj}(y), \vee_{j\in J}\ \nu_{Aj}(y)\rangle$
(2) $\cup A_j$ may be defined as:
(i) $\cup A_j = \langle y, \vee_{j\in J}\ \mu_{Aj}(y), \vee_{j\in J}\ \sigma_{Aj}(y), \wedge_{j\in J}\ \nu_{Aj}(y)\rangle$
(ii) $\cup A_j = \langle y, \vee_{j\in J}\ \mu_{Aj}(y), \wedge_{j\in J}\ \sigma_{Aj}(y), \wedge_{j\in J}\ \nu_{Aj}(y)\rangle$

Proposition 2.10: [10]

For all A and B are two neutrosophic sets then the following conditions are true:
$C(A\cap B) = C(A) \cup C(B)$; $C(A\cup B) = C(A) \cap C(B)$.

Definition 2.11: [10]

A Neutrosophic topology [NT for short] is a non-empty set Y is a family τ of neutrosophic subsets in Y satisfying the following axioms:

(NT_1) 0_N, $1_N \in \tau$,
(NT_2) $G_1 \cap G_2 \in \tau$ for any G_1, $G_2 \in \tau$,
(NT_3) $\cup G_i \in \tau$ for every $\{ G_i : i \in J\} \subseteq \tau$.

Throughout this paper, the pair (Y, τ) is called a neutrosophic topological space (NTS for short). The elements of τ are called neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (Y, τ) is called a neutrosophic closed set [NCS for short] in Y.

Example 2.12: [10]

Any fuzzy topological space (Y, τ_0) in the sense of Chang is obviously a NTS in the form $\tau = \{A: \mu_A \in \tau_0\}$ wherever we identify a fuzzy set in Y whose membership function is μ_A with its counterpart.

The following is an example of Neutrosophic topological space.

Example 2.13: [10]

Let Y = {y} and A = {⟨y, 0.5, 0.5, 0.4⟩: y ∈ Y}, B = {⟨y, 0.4, 0.6, 0.8⟩: y ∈ Y}, C = {⟨y, 0.5, 0.6, 0.4⟩: y ∈ Y}, D = {⟨y, 0.4, 0.5, 0.8⟩: y ∈ Y}. Then the family $\tau = \{0_N, A, B, C, D, 1_N\}$ of NSs in Y is neutrosophic topological space on Y.

Now, we define the neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces:

Definition 2.14: [10]

Let (Y, τ) be NTS and $A = \{\langle y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle: y \in Y\}$ be a NS in Y. Then the neutrosophic closure and neutrosophic interior of A are defined by
NCl(A) = ∩{K : K is a NCS in Y and A ⊆ K}
NInt(A) = ∪{G : G is a NOS in Y and G ⊆ A}
It can be also shown that *NCl*(A) is NCS and *NInt*(A) is a NOS in Y.

a) A is NOS if and only if A = *NInt*(A),
b) A is NCS if and only if A = *NCl*(A).

Proposition 2.15: [10]

For any neutrosophic set A is (Y, τ) we have

a) *NCl*(C(A)) = C(*NInt*(A)),
b) *NInt*(C(A)) = C(*NCl*(A)).

Proposition 2.16: [10]

Let (Y, τ) be NTS and A, B be two neutrosophic sets in Y. Then the following properties are holds:

a) *NInt*(A) ⊆ A,
b) A ⊆ *NCl*(A),
c) A ⊆ B ⇒ *NInt*(A) ⊆ *NInt*(B),
d) A ⊆ B ⇒ *NCl*(A) ⊆ *NCl*(B),
e) *NInt*(*NInt*(A)) = *NInt*(A),
f) *NCl*(*NCl*(A)) = *NCl*(A),
g) *NInt*(A ∩ B) = *NInt*(A) ∩ *NInt*(B),
h) *NCl*(A ∪ B) = *NCl*(A) ∪ *NCl*(B),
i) *NInt*(0_N) = 0_N,
j) *NInt*(1_N) = 1_N,
k) *NCl*(0_N) = 0_N,
l) *NCl*(1_N) = 1_N,
m) A ⊆ B ⇒ C(A) ⊆ C(B),
n) *NCl*(A ∩ B) ⊆ *NCl*(A) ∩ *NCl*(B),
o) *NInt*(A ∪ B) ⊇ *NInt*(A) ∪ *NInt*(B).

Definition 2.17: [5]

A NS $A = \{\langle y, \mu_A(y), \sigma_A(y), \nu_A(y)\rangle: y \in Y\}$ in a NTS (Y, τ) is said to be

(i) *Neutrosophic regular closed set* (NRCS for short) if A = *NCl*(*NInt*(A)),
(ii) *Neutrosophic regular open set* (NROS for short) if A = *NInt*(*NCl*(A)),
(iii) *Neutrosophic semi closed set* (NSCS for short) if *NInt*(*NCl*(A)) ⊆ A,

(iv) *Neutrosophic semi open set* (NSOS for short) if A ⊆ *NCl*(*NInt*(A)),
(v) *Neutrosophic pre closed set* (NPCS for short) if *NCl*(*NInt*(A)) ⊆ A,
(vi) *Neutrosophic pre open set* (NPOS for short) if A ⊆ *NInt*(*NCl*(A)).
(vii) *Neutrosophic α- closed set* (NSCS for short) if *NCl*(*NInt*(*NCl*(A))) ⊆ A,
(viii) *Neutrosophic α- open set* (NSOS for short) if A ⊆ *NInt*(*NCl*(*NInt*(A))),

Definition 2.18: [14]

Let (Υ, τ) be NTS and A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} be a NS in Υ. Then the neutrosophic pre closure and neutrosophic pre interior of A are defined by

NPCl(A) = ∩{K : K is a NPCS in Υ and A ⊆ K},
NPInt(A) = ∪{G : G is a NPOS in Υ and G ⊆ A}.

Definition 2.19: [9]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic semi-pre closed set (NSPCS for short) if and only if Nint(Ncl(Nint(A))) ⊆ A. A NS A of a NTS (Υ, τ) is called a neutrosophic semi-pre open set (NSPOS for short)) if and only if A ⊆ Nint(Ncl(Nint(A))).

Definition 2.20. [9]

Let A be a Neutrosophic set in Neutrosophic topology (Υ, *τ*). Then is Neutrosophic semi pre interior of A [NSPint(A)] and Neutrosophic semi pre closure of A [NSPCI (A)] are defined by

NSP int (A) = ∪{G : G is a NSPOS in Υ and G ⊆A},
NSPcl (A) = ∩{K : K is a NSPCS in Υ and A ⊆K}.

Definition 2.21: [7]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic generalized closed set (NGCS for short) if *NCl*(A) ⊆ U whenever A ⊆ U and U is a NOS in (Υ, τ). A NS A of a NTS (Υ, τ) is called a neutrosophic generalized open set (NGOS for short) if C(A) is a NGCS in (Υ, τ).

Definition 2.22: [12]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic *ω* closed set (N*ω*CS for short) if *NCl*(A) ⊆ U whenever A ⊆ U and U is a NSOS in (Υ, τ). A NS A of a NTS (Υ, τ) is called a neutrosophic *ω* open set (N*ω*OS for short) if C(A) is a N*ω*CS in (Υ, τ).

Definition 2.23: [5]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic regular generalized closed set (NRGCS for short) if *NCl*(A) ⊆ U whenever A ⊆ U and U is a NROS in (Υ, τ). A NS A of a NTS (Υ, τ) is called a neutrosophic regular generalized open set (NRGOS for short) if C(A) is a NRGCS in (Υ, τ).

Definition 2.24: [14]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic generalized pre closed set (NGPCS for short) if *NPCl*(A) ⊆ U whenever A ⊆ U and U is a NOS in (Υ, τ). A NS A of a NTS (Υ, τ) is called a neutrosophic generalized pre open set (NGPOS for short) if C(A) is a NGPCS in (Υ, τ).

Definition 2.25: [8]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic generalized semipre closed set (NGSPCS for short) if *NSPCl*(A) ⊆ U whenever A ⊆ U and U is a NOS in (Υ, τ). A NS A of a NTS (Υ, τ) is called a neutrosophic generalized semipre open set (NGSPOS for short) if C(A) is a NGSPCS in (Υ, τ).

Definition 2.26: [6]

A NS A = {⟨y, μ_A(y), σ $_A$(y), ν_A(y)⟩: y ∈ Υ} in a NTS (Υ, τ) is said to be a neutrosophic generalized pre regular closed set (NGPRCS for short) if *NPCl*(A) ⊆ U whenever A ⊆ U and U is a NROS in (Υ, τ). A NS A of a NTS (Υ, τ) is called a neutrosophic generalized pre regular open set (NGPROS for short) if C(A) is a NGPRCS in (Υ, τ).

3 Neutrosophic Generalized SPR Closed Sets

Definition 3.1:

A NS A in a NTS (ϒ, τ) is said to be a neutrosophic generalized SPR closed set (NGSPRCS for short) if *NSPCl*(A) ⊆ U whenever A ⊆ U and U is a NROS in (ϒ, τ). The family of all NGSPRCSs of a NTS(ϒ, τ) is denoted by NGSPRC(ϒ).

Example 3.2: Let ϒ= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.2, 0.2, 0.6), (0.1, 0.4, 0.8)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N and 1_N is a NROS, we have *NSPCl*(A) = A ⊆1_N.

Theorem 3.3: Every NCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NCS in (ϒ, τ), we have *NCl* (A) = A. Therefore *NSPCl*(A) ⊆ *NCl* (A) = A ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.4: In Example 3.2., the NS A= A= ⟨(0.2, 0.2, 0.6), (0.1, 0.4, 0.8)⟩ is a NGSPRCS but not NCS in (ϒ, τ).

Theorem 3.5: Every NαCS in (ϒ, τ) is an NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NαCS in (ϒ, τ), we have *NCl(NInt(NCl(A)))* ⊆ A, now *NInt*(A) ⊆ *A*, *NInt(NCl(NInt(A)))* ⊆ *NCl(NInt(NCl(A)))* ⊆ A. Therefore *NSPCl*(A) = A∪ *NInt(NCl(NInt(A)))* ⊆ A∪A = A ⊆ U. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.6: Let ϒ= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V = ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.7, 0.2, 0.3), (0.8, 0.2, 0.2)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N and 1_N is a NROS, we have *NSPCl*(A) = A ⊆ 1_N. But A is not NαCS in (ϒ, τ). We have *NCl*(*NInt(NCl(A)))* = 1_N ⊈ A.

Theorem 3.7: Every NωCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NωCS in (ϒ, τ), we have *NCl* (A) ⊆ U because every NROS is NSOS in (ϒ, τ). Therefore *NSPCl*(A) ⊆ *NCl* (A) ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.8: Let ϒ= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.2, 0.2, 0.6), (0.1, 0.4, 0.8)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N and 1_N is a NROS, we have *NSPCl*(A) = A ⊆1_N. But A is not NωCS in (ϒ, τ). Since A ⊆ A and A is a NSOS, we have *NCl*(A) = 1_N ⊈ A.

Theorem 3.9: Every NPCS in (ϒ, τ) is an NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NPCS in (ϒ, τ), we have *NCl(NInt(A))* ⊆ A. Therefore *NSPCl(A)* ⊆*NPCl*(A) = A∪ *NCl(NInt(A))* ⊆ A∪A = A ⊆ U. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.10: Let ϒ= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.8, 0.4, 0.3), (0.2, 0.4, 0.7)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.8, 0.5, 0.2), (0.3, 0.4, 0.6)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N, we have *NSPCl*(A) = 1_N ⊆ 1_N. But A is not NPCS in (ϒ, τ). Since *NCl(NInt(A))* = 1_N ⊈ A.

Theorem 3.11: Every NGCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NGCS in (ϒ, τ) and every NROS in (ϒ, τ) is a NOS in (ϒ, τ). Therefore *NSPCl*(A) ⊆ *NCl* (A) ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.12: Let ϒ= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V = ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.7, 0.2, 0.3), (0.8, 0.2, 0.2)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N and 1_N is a NROS, we have *NSPCl*(A) = A ⊆ 1_N. But A is not NGCS in (ϒ, τ). Since A ⊆ V and V is a NOS, we have *NCl*(A) = 1_N ⊈ V.

Theorem 3.13: Every NSCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NSCS in (ϒ, τ). Therefore *NSPCl*(A) ⊆ *NSCl* (A) = A ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.14: In Example 3.12., the NS A= ⟨(0.7, 0.2, 0.3), (0.8, 0.2, 0.2)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N and 1_N is a NROS, we have *NSPCl*(A) = A ⊆ 1_N. But A is not NSCS in (ϒ, τ). Since *NInt(NCl(A))* = 1_N ⊈ A.

Theorem 3.15: Every NSPCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NSPCS in (ϒ, τ). Therefore *NSPCl*(A) = *A* ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.16: Let ϒ= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.5, 0.4, 0.2), (0.8, 0.6, 0.6)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N, we have *NSPCl*(A) = 1_N ⊆ 1_N. But A is not NSPCS in (ϒ, τ). Since *NInt(NCl(NInt(A)))* = 1_N ⊈ A.

Theorem 3.17: Every NGPCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NGPCS in (ϒ, τ) and every NROS in (ϒ, τ) is a NOS in (ϒ, τ). Therefore *NSPCl(A)* ⊆ *NPCl(A)* ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.18: Let ϒ= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.8, 0.4, 0.2), (0.7, 0.5, 0.1)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.8, 0.4, 0.2), (0.7, 0.5, 0.1)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N, we have *NSPCl*(A) = 1_N ⊆ 1_N. But A is not NGPCS in (ϒ, τ). Since A ⊆ U and U is a NOS, we have *NPCl*(A) = 1_N ⊈ U.

Theorem 3.19: Every NRGCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NRGCS in (ϒ, τ). Therefore *NSPCl(A)* ⊆ *NPCl*(A) ⊆ *NCl* (A) ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.20: Let ϒ= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.4, 0.7), (0.4, 0.4, 0.6)⟩ and V = ⟨(0.8, 0.4, 0.2), (0.7, 0.5, 0.1)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.3, 0.2, 0.7), (0.2, 0.3, 0.8)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ U and U is a NROS, we have *NSPCl*(A) = A ⊆ U. But A is not NRGCS in (ϒ, τ). Since A ⊆ U and U is a NROS, we have *NCl*(A) = C(U) ⊈ U.

Theorem 3.21: Every NGSPCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NGSPCS in (ϒ, τ) and every NROS in (ϒ, τ) is a NOS in (ϒ, τ). Therefore *NSPCl(A)* ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

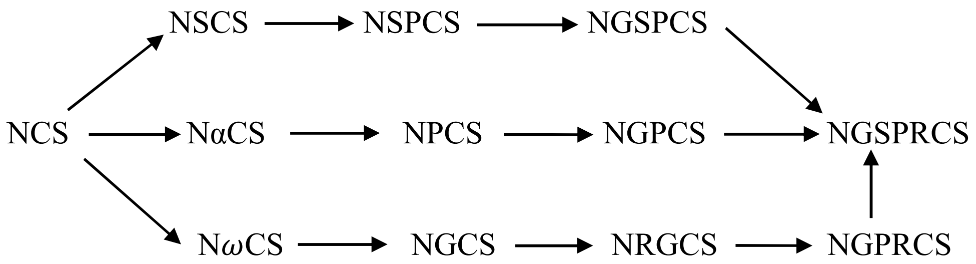
Example 3.22: Let ϒ= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.7, 0.8, 0.4), (0.4, 0.3, 0.3)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.7, 0.8, 0.4), (0.4, 0.3, 0.3)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ 1_N, we have *NSPCl*(A) = 1_N ⊆ 1_N. But A is not NGSPCS in (ϒ, τ). Since A ⊆ U and U is a NOS, we have *NSPCl*(A) = 1_N ⊈ U.

Theorem 3.23: Every NGPRCS in (ϒ, τ) is a NGSPRCS in (ϒ, τ) but not conversely.

Proof: Let U be a NROS in (ϒ, τ) such that A ⊆ U. Since A is NGPRCS in (ϒ, τ). Therefore *NSPCl(A)* ⊆ *NPCl(A)* ⊆ U, by hypothesis. Hence A is a NGSPRCS in (ϒ, τ).

Example 3.24: Let ϒ= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩**.** Then (ϒ, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ is a NGSPRCS in (ϒ, τ). Since A ⊆ U and U is a NROS, we have *NSPCl*(A) = A ⊆ U. But A is not NGPRCS in (ϒ, τ). Since A ⊆ U and U is a NROS, we have *NPCl*(A) = C(U) ⊈ U.

The following diagram, we have provided the relation between NGSPRCS and the other existed NSs.



In this diagram by "A $\longrightarrow$ B" means A implies B but not conversely and A $\nleftrightarrow$ B means A & B are independent.

Theorem 3.25: Let (Y, τ) be a NTS. Then for every A ∈ NGSPRC(Y) and for every NS B ∈ NS(Y), A ⊆ B ⊆ *NSPCl*(A) implies B ∈ NGSPRC(Y).

Proof: Let B ⊆ U and U is a NROS in (Y, τ). Since A ⊆ B, then A ⊆ U. Given A is a NGSPRCS, it follows that *NSPCl*(A) ⊆ U. Now B ⊆ *NSPCl*(A) implies *NSPCl*(B) ⊆ *NSPCl*(*NPCl*(A)) = *NSPCl*(A). Thus, *NSPCl*(B) ⊆ U. This proves that B ∈ NGSPRC(Y).

Theorem 3.26: If A is a NROS and a NGSPRCS in (Y, τ), then A is a NSPCS in (Y, τ).

Proof: Since A ⊆ A and A is a NROS in (Y, τ), by hypothesis, *NSPCl*(A) ⊆ A. But since A ⊆ *NSPCl*(A). Therefore *NSPCl*(A)= A. Hence A is a NSPCS in (Y, τ).

Theorem 3.27: Let (Y, τ) be a NTS and NSPC(Y) (resp. NRO(Y)) be the family of all NSPCSs (resp. NROSs) of Y. If NSPC(Y) = NRO(Y) then every neutrosophic subset of Y is NGSPRCS in (Y, τ).

Proof: If NSPC(Y) = NRO(Y) and A is any neutrosophic subset of Y such that A ⊆ U where U is NROS in Y. Then by hypothesis, U is NSPCS in Y which implies that *NSPCl*(U) = U. Then *NSPCl*(U) ⊆ *NSPCl*(U) = U. Therefore A is NGSPRCS in (Y, τ).

Definition 3.28:

Let (Y, τ) be a NTS and A = {⟨y, μ_A(y), σ_A(y), ν_A(y)⟩: y ∈ Y} be the subset of Y. Then
NGSPRCl(A) = ∩{K : K is a NGSPRCS in Y and A ⊆ K} and
NGSPRInt(A) = ∪{G : G is a NGSPROS in Y and G ⊆ A}.

Lemma 3.29:

Let A and B be subsets of (Y, τ). Then the following results are obvious.

a) *NGSPRCl*(0_N) = 0_N.
b) *NGSPRCl*(1_N) = 1_N.
c) A ⊆ *NGSPRCl*(A).
d) A ⊆ B ⇒ *NGSPRCl*(A) ⊆ *NGSPRCl*(B).

4 Neutrosophic Generalized SPR Open Sets

Definition 4.1:

A NS A in a NTS (Y, τ) is said to be a neutrosophic generalized SPR open set (NGSPROS for short) if *NSPInt*(A) ⊇ U whenever A ⊇ U and U is a NRCS in (Y, τ). Alternatively, A NS A is said to be a neutrosophic generalized SPR open set (NGSPROS for short) if the complement of C(A) is a NGSPRCS in (Y, τ).
The family of all NGSPROSs of a NTS(Y, τ) is denoted by NGSPRO(Y).

Example 4.2: Let Y= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.6, 0.8, 0.2), (0.8, 0.6, 0.1)⟩ is a NGSPROS in (Y, τ).

Theorem 4.3: Every NOS is a NGSPROS in (Y, τ) but the converses may not be true in general.

Proof: Let U be a NRCS in (Y, τ) such that A ⊒ U. Since A is NOS, *NInt*(A) = A. By hypothesis, *NSPInt*(A) ⊒ *NPInt*(A) = A ∩ *NInt*(*NCl*(A)) = A ∩ *NCl*(A) ⊒ A ∩ A = A ⊒ U. Therefore A is a NGSPROS in (Y, τ).

Example 4.4: In Example 4.2., the NS A= ⟨(0.6, 0.8, 0.2), (0.8, 0.6, 0.1)⟩ is an NGSPROS in (Y, τ) but not a NOS in (Y, τ).

Theorem 4.5: Every NαOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.6: Let Y= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V = ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.3, 0.8, 0.7), (0.2, 0.8, 0.8)⟩ is a NGSPROS in (Y, τ). But A is not NαOS in (Y, τ).

Theorem 4.7: Every NωOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.8: Let Y= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.6, 0.8, 0.2), (0.8, 0.6, 0.1)⟩ is a NGSPROS in (Y, τ). But A is not NωOS in (Y, τ).

Theorem 4.9: Every NPOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.10: Let Y= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.8, 0.4, 0.3), (0.2, 0.4, 0.7)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.2, 0.5, 0.8), (0.6, 0.6, 0.3)⟩ is a NGSPROS in (Y, τ). But A is not NPOS in (Y, τ).

Theorem 4.11: Every NSOS and NGOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.12: Let Y= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V = ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.3, 0.8, 0.7), (0.2, 0.8, 0.8)⟩ is a NGSPROS in (Y, τ). But A is not NSOS and NGOS in (Y, τ).

Theorem 4.13: Every NSPOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.14: Let Y= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.2, 0.6, 0.5), (0.6, 0.4, 0.8)⟩ is a NGSPROS in (Y, τ). But A is not NSPOS in (Y, τ).

Theorem 4.15: Every NGPOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.16: Let Y= {a, b} and τ = {0_N, U, 1_N} where U= ⟨(0.8, 0.4, 0.2), (0.7, 0.5, 0.1)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.2, 0.6, 0.8), (0.1, 0.5, 0.7)⟩ is a NGSPROS in (Y, τ). But A is not NGPOS in (Y, τ).

Theorem 4.17: Every NRGOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.18: Let Y= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.4, 0.7), (0.4, 0.4, 0.6)⟩ and V = ⟨(0.8, 0.4, 0.2), (0.7, 0.5, 0.1)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.7, 0.8, 0.3), (0.1, 0.5, 0.7)⟩ is a NGSPROS in (Y, τ). But A is not NRGOS in (Y, τ).

Theorem 4.19: Every NGSPOS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.20: Let Y= {a, b} and τ = {0_N, U, 1_N} where ⟨(0.7, 0.8, 0.4), (0.4, 0.3, 0.3)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.4, 0.2, 0.7), (0.3, 0.7, 0.4)⟩ is a NGSPROS in (Y, τ). But A is not NGSPOS in (Y, τ).

Theorem 4.21: Every NGPROS is a NGSPROS in (Y, τ) but the converses are not true in general.

Example 4.22: Let Y= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (Y, τ) is a neutrosophic topological space. Here the NS A= ⟨(0.6, 0.7, 0.5), (0.7, 0.6, 0.4)⟩ is a NGSPROS in (Y, τ). But A is not NGPROS in (Y, τ).

Theorem 4.23: Let (Y, τ) be a NTS. Then for every A ∈ NGSPRO(Y) and for every B ∈ NS(Y), *NSPInt*(A) ⊆ B ⊆ A implies B ∈ NGSPRO(Y).

Proof: Let A be any NGSPROS of (Y, τ) and B be any NS of Y. By hypothesis *NSPInt*(A) ⊆ B ⊆ A. Then C(A) is an NGSPRCS in (Y, τ) and C(A) ⊆ C(B) ⊆ *NSPCl*(C(A)). By **Theorem 3.25.,** C(B) is an NGSPRCS in (Y, τ). Therefore B is an NGSPROS in (Y, τ). Hence B ∈ NGSPRO(Y).

Theorem 4.24: A NS A of a NTS (Y, τ) is a NGSPROS in (Y, τ) if and only if F ⊆ *Nspint*(A) whenever F is a NRCS in (Y, τ) and F ⊆ A.

Proof: Necessity: Suppose A is a NGSPROS in (Y, τ). Let F be a NRCS in (Y, τ) such that F ⊆ A. Then C(F) is a NROS and C(A) ⊆ C(F). By hypothesis CA) is a NGSPRCS in (Y, τ), we have *NSPCl*(C(A)) ⊆ C(F). Therefore F ⊆ *Nspint*(A).
Sufficiency: Let U be a NROS in (Y, τ) such that C(A) ⊆ U. By hypothesis, C(U) ⊆ *Nspint*(A). Therefore *NSPCl*(C(A)) ⊆ U and C(A) is a NGSPRCS in (Y, τ). Hence A is a NGSPROS in (Y, τ).

Theorem 4.25: Let (Y, τ) be a NTS and NSPO(Y) (resp. NGSPRO(Y)) be the family of all NSPOSs (resp. NGSPROSs) of Y. Then NSPO(Y) ⊆ NGSPRO(Y).

Proof: Let A ∈ NSPO(Y). Then C(A) is NSPCS and so NGSPRCS in (Y, τ). This implies that A is NGSPROS in (Y, τ). Hence A ∈ NGSPRO(Y). Therefore NSPO(Y) ⊆ NGSPRO(Y).

5 Separation Axioms of Neutrosophic Generalized SPR Closed Sets

Definition 5.1:

If every NGSPRCS in (Y, τ) is a NSPCS in (Y, τ), then the space (Y, τ) can be called a neutrosophic $SPRT_{1/2}$ ($NSPRT_{1/2}$ for short) space.

Theorem 5.2: An NTS (Y, τ) is a $NSPRT_{1/2}$ space if and only if NSPOS(Y) = NGSPRO(Y).

Proof: Necessity: Let (Y, τ) be a $NSPRT_{1/2}$ space. Let A be a NGSPROS in (Y, τ). By hypothesis, C(A) is a NGSPRCS in (Y, τ) and therefore A is a NSPOS in (Y, τ). Hence NSPO(Y) = NGSPRO(Y).
Sufficiency: Let NSPO(Y) = NGSPRO(Y). Let A be a NGSPRCS in (Y, τ). Then C(A) is a NGSPROS in (Y, τ). By hypothesis, C(A) is a NSPOS in (Y, τ) and therefore A is a NSPCS in (Y, τ). Hence (Y, τ) is a $NSPRT_{1/2}$ space.

Definition 5.3: A NTS (Y, τ) is said to be a neutrosophic $SPRT^*_{1/2}$ space ($NSPRT^*_{1/2}$ space for short) if every NGSPRCS is a NCS in (Y, τ).

Remark 5.4: Every $NSPRT^*_{1/2}$ space is a $NSPRT_{1/2}$ space.

Proof: Assume (Y, τ) is a $NSPRT^*_{1/2}$ space. Let A be a NGSPRCS in (Y, τ). By hypothesis, A is an NCS. Since every NCS is a NSPCS, A is a NSPCS in (Y, τ). Hence (Y, τ) is a $NSPRT_{1/2}$ space.

Example 5.5: Let Y= {a, b} and let τ = {0_N, U, 1_N} where U= ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (Y, τ) is a $NSPRT_{1/2}$ space, but it is not $NSPRT^*_{1/2}$ space. Here the NS A= ⟨(0.2, 0.3, 0.8), (0.3, 0.4, 0.8)⟩ is a NGSPRCS but not a NCS in (Y, τ).

Theorem 5.6: Let (Y, τ) be a $NSPRT^*_{1/2}$ space then,
(i) the union of NGSPRCSs is NGSPRCS in (Y, τ)
(ii) the intersection of NGSPROSs is NGSPROS in (Y, τ)

Proof: (i) Let $\{A_i\}_{i\in J}$ be a collection of NGSPRCSs in a $NSPRT^*_{1/2}$ space (Y, τ). Thus, every NGSPRCSs is a NCS. However, the union of NCSs is a NCS in (Y, τ). Therefore the union of NGSPRCSs is NGSPRCS in (Y, τ).
(iii) proved by taking the complement in (i).

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.

[2] C. L. Chang, Fuzzy topological spaces, J.Math.Anal.Appl., 24 (1968), 182- 190.

[3] Dogan Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1) (1997), 81-89.

[4] Floretin Smarandache, Neutrosophic Set:- A Generalization of Intuitionistiic Fuzzy set, Journal of Defense Resourses Management, 1(2010),107–116.

[5] P. Ishwarya and K. Bageerathi, On Neutrosophic semi-open sets in neutrosophic topological spaces, International Jour. of Math. Trends and Tech, (2016), 214-223

[6] I. Mohammed Ali Jaffer and K. Ramesh, Neutrosophic Generalized Pre Regular Closed Sets, Neutrosophic Sets and Systems, 30(2019), 171-181.

[7] A. Pushpalatha and T. Nandhini, Generalized closed sets via neutrosophic topological Spaces, Malaya Journal of Matematik, 7(1) (2019), 50-54.

[8] N. Rajeshwaran and N. Chandramathi, Generalized Semi Pre Closed Sets via Neutrosophic Topological Space, Strad Research, 8(12) (2021), 174 – 180.

[9] Renu Thomas and Anila. S, On Neutrosophic Semi-preopen Sets and Semi-preclosed Sets in a Neutrosophic Topological Space, International Journal of Scientific Research in Mathematical and Statistical Sciences, 5(5) (2018), 138-143.

[10] A. A. Salama and S. A. Alblowi, Neutrosophic set and Neutrosophic topological spaces, IOSR Jour. of Mathematics, (2012), 31-35.

[11] A. A. Salama, Florentin Smarandache and Valeri Kroumov, Neutrosophic Closed set and Neutrosophic Continuous Function, Neutrosophic Sets and Systems, 4(2014), 4–8.

[12] R. Santhi and N. Udhayarani, Nω –Closed sets in Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, 12 (2016), 114-117.

[13] V. Venkateswara Rao and Y. Srinivasa Rao, Neutrosophic Pre-open sets and Pre-closed sets in Neutrosphic Topology, International Jour. ChemTech Research, 10(10) (2017), 449-458.

[14] Wadei Al-Omeri and Saeid Jafari, On Generalized Closed Sets and Generalized Pre-Closed in Neutrosophic Topological Spaces, Mathematics MDPI, 7(1) (2018), 01-12.

[15] L. A. Zadeh, Fuzzy sets, Information and control, 8(1965), 338-353.